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Quark Interpretation of the Combinatorial Hierarchy

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We propose a physical interpretation of the second level of the combinatorial hierarchy in terms of three quarks, three antiquarks, and the vacuum. This interpretation allows us to introduce a new quantum number, which measures electromagnetic mass splitting of the quarks. We extend our argument by analog to baryons, and find some $SU(3)$ and some new mass formulas for baryons. The generalization of our approach to other hierarchy levels is discussed. We present also an empirical mass formula for baryons, which seems to be loosely connected with the combinatorial hierarchy.

1. INTRODUCTION

The combinatorial hierarchy is a mathematical structure recently investigated by Ted Bastin, H. Pierre Noyes, John Amson, and Clive W. Kilminster (Bastin and Noyes, 1978; Noyes, 1979; Bastin et al., 1979), who attempted to interpret it in terms of elementary particle systems. The combinatorial hierarchy is supposed to represent a classification scheme for states of matter, and each level of the hierarchy is believed to be connected with different class of interactions, together combining strong, electromagnetic, and gravitational interactions into a single picture. We shall review for convenience the main properties of the combinatorial hierarchy.

Consider the group of N-tuples $x=(x_1, x_2,\ldots, x_N)$ with $x_k \in Z_2=\{0,1\}$ under addition, modulo 2. *Discrimination* D between elements x and y is defined as

$$D(x, y)=(x_1, x_2,\ldots, x_N)+(y_1, y_2,\ldots, y_N)$$
$$=(x_1+y_1, x_2+y_2,\ldots, x_N+y_N) \tag{1.1}$$

A subset $S \subseteq Z_2^N$ is said to be *discriminately closed* if for all $x, y \in S$, if $x \neq y$ then $D(x, y) \in S$, that is, if by adding two N-tuples belonging to S we always end up with a third different N-tuple also belonging to S. Note that the *neutral element* $e_0 = (0, 0, \ldots, 0)$ does not belong to any discriminately closed subset (DCsS for short). The act of discrimination between two N-tuples can be viewed as a method by which the quantum numbers of two physical systems can be compared at a (supposedly prespaciotemporal) locus.

However, the act of discrimination as such and the existence of discriminately closed sets, even if somehow applicable to the realm of elementary particles in some abstract form, would perhaps not be of great interest. The physical relevance of the above presented simple systems lies in the fact that it is possible to construct a finite sequence of hierarchial systems of increasing complexity, which together are called *the combinatorial hierarchy*. It consists of four levels. The first level is represented by columns of dimension $n=2$; explicitly they are $e_1 = \binom{1}{0}$, $e_2 = \binom{0}{1}$, and $e_3 = \binom{1}{1}$. It is obvious that out of these elements can be formed three discriminately closed subsets, namely, sets $\{e_1\}$, $\{e_2\}$, and $\{e_1, e_2, e_1 + e_2 = e_3\}$. It is also easy to verify that generally for N basis elements, $2^N - 1$ DCsS can be formed. In this context the term "basis elements" refers to elements which through successive discrimination generate all the other elements of the discriminate system under consideration. For example, the basis elements at the first level of the combinatorial hierarchy are e_1 and e_2. Naturally the basis elements can be chosen in many ways.

The next (second) level of the combinatorial hierarchy is obtained by taking the three DCsS of the first level to form the basis elements of the second level. Then at the second level there are $2^3 - 1 = 7$ elements as well as DCsS. These in their turn serve as the basis for a third level, where $2^7 - 1 = 127$ DCsS can be formed. Finally, at the fourth level there are 127 basis elements and $2^{127} - 1 \sim 10^{38}$ DCsS.

If $2^N - 1$ DCsS are to form the basis of the next level, it is convenient to map them to columns of some height. In practice this is done by $2^N - 1$ nonsingular (in order to preserve the structure of the preceding level), linearly independent matrices, which map each column in one of the DCsS onto itself. The elements of these matrices can then be rearranged into

TABLE I. Some properties of different levels of the combinatorial hierarchy.

Level	I	II	III	IV
No. of basis elements	2	3	7	127
No. of DCsS	3	7	127	$\sim 10^{38}$
Dimension of elements	2	4	16	256
Cumulative sum of DCsS	3	10	137	$\sim 10^{38}$

columns, (by some convention) of height n^2, if the height of the columns in the preceding level is n. Note that this process terminates if the number of DCsS, $2^N - 1$, is greater than the number of elements (n^2) in the mapping matrices, since for matrices of dimension n there are only n^2 that are linearly independent. This is why the fifth level of the hierarchy cannot be reached: there are only $(256)^2$ linearly independent matrices available, while the number of DCsS was seen to be 10^{38}. Since the preceding level is in a sense contained in the following level, the characteristic cardinal of each level is the cumulative sum of the inverse values of superstrong, strong, electromagnetic, and gravitational coupling constants can be seen. These values, together with other numerology, are collected in Table I.

2. PHYSICAL INTERPRETATION OF THE SECOND LEVEL

We now turn our attention to the second level. It can be shown (Bastin and Noyes, 1978), that the second level will always be, up to a permutation of rows

$$\begin{pmatrix}1\\1\\0\\0\end{pmatrix}\begin{pmatrix}1\\1\\0\\1\end{pmatrix}\begin{pmatrix}1\\1\\1\\0\end{pmatrix}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}\begin{pmatrix}0\\0\\0\\1\end{pmatrix}\begin{pmatrix}0\\0\\1\\0\end{pmatrix}\begin{pmatrix}0\\0\\1\\1\end{pmatrix} \quad (2.1)$$

Bastin and Noyes interpret the values of elements in each column to represent presence (=1) or absence (=0) of some quantum number. Specifically they interpret the second level as bosonic systems, like baryon –antibaryon pairs or mesons.

We propose an alternative interpretation which, in our view, results in a more consistent picture, and besides, leads to suggestive concrete predictions. Bastin and Noyes, in their interpretation of the elements of columns as representatives of physically observable quantum numbers, are naturally led to regard the discarded column (0, 0, 0, 0) as the vacuum. However, this needs not to be the case. In our view, the elements of the hierarchy give the possible *names* of some abstract internal properties of space–times domains (or points), independently whether these domains exist or not. The internal labels of these names, which characterize the names and differentiate them for each other, need not be physically observable; neither need they represent any abstract quantum number. In fact, they do not even exist independently but only in relation to labels in other names, and they are made meaningful only through the act of discrimination. On the other hand, *if* the hierarchy is supposed to represent internal properties of space–time domains, it certainly would be reasonable to expect "vacuumness" to be included in the hierarchy. Moreover, fermionic systems are more elementary than bosonic in the sense that bosonic systems can be build up from fermionic systems but not vice versa. Guided by these considerations we are led to try to interpret the second level in terms of elementary fermions, i.e., quarks and antiquarks, and the vacuum.

As can be seen from equation (2.1), the first two elements in the columns are always $\binom{1}{1}$ or $\binom{0}{0}$. This fact can be viewed as a kind of reduction of dimension, since the second level happens to be isomorphic to first level of another hierarchy beginning with columns of dimension *three*. The three-dimensional first level is given by

$$e_1=\begin{bmatrix}1\\0\\0\end{bmatrix},\quad e_2=\begin{bmatrix}0\\1\\0\end{bmatrix},\quad e_3=\begin{bmatrix}0\\0\\1\end{bmatrix},\quad e_4=\begin{bmatrix}0\\1\\1\end{bmatrix},\quad e_5=\begin{bmatrix}1\\0\\1\end{bmatrix},$$
$$e_6=\begin{bmatrix}1\\1\\0\end{bmatrix},\quad e_7=\begin{bmatrix}1\\1\\1\end{bmatrix} \tag{2.2}$$

A hierarchial system can be constructed by starting from this level. The next level is nine-dimensional, and there this hierarchy terminates. For the sake of simplicity, we choose to work with the first level of the three-dimensional representation rather than with the four-dimensional second level. However, this choice in no way affects the results we shall obtain.

Our interpretation is the following. We choose basis elements e_1, e_2, and e_3 to represent quarks u_h, d_h, and s_h, respectively, where the subscript h indicates that the quarks are defined within the hierarchy. We define antiquarks as boolean duals of quarks, that is, if q_h is a quark, then the respective antiquark is $\bar{q}_h=q_h+e_7$, where the addition is modulo 2, whence $\bar{u}_h=e_4$, $\bar{d}_h=e_5$, and $\bar{s}_h=e_6$. The seventh element, e_7, is then the name of the vacuum. This is also reflected by the fact that $D(q_h, \bar{q}_h)=e_7$, which, perhaps, indicates the possibility of a quark–antiquark annihilation.

Physical quarks are, however, labeled by the values of physically observable quantum numbers (which, among other things, do not obey modulo 2 algebra). Physical vacuum is always represented by the null element in the space spanned by generators of quark quantum numbers. The problem we are faced with is that of connecting the names of the quark states in the hierarchy to quantum number labels of physical quarks. In doing this, we shall assume that the information content of the names of the hierarchy states is maximal, or in other words, that for every component of columns in the hierarchy there exists (but not necessarily corresponds to) a dimension in the physical quantum number space. Thus every element of a column belonging to the hierarchy carries information about three different quantum numbers. We must only rename every label.

Physical quarks are labeled by their hypercharge Y and third component of their isospin. According to our proposal, the hierarchy quarks carry a piece of their total Y or I_3 in every component of the column corresponding to the hierarchy quark. The total quantum numbers of the vacuum must be zero. Effectively, the renaming of the hierarchy labels can be done with a suitable 3×3 matrix R which transforms the hierarchy labels into physical labels. We write R as

$$R=\begin{bmatrix} I_{3,1} & I_{3,2} & I_{3,3} \\ Y_1 & Y_2 & Y_3 \\ C_1 & C_2 & C_3 \end{bmatrix} \tag{2.3}$$

C represents a property thus far undefined. In order that the vacuum e_7 would be transformed to the physical vacuum $(0,0,0)$ the sum of every row in equation (2.3) must be zero. Note that this defines the geometry of the quark quantum number space; only the scale remains arbitrary [as it does in $SU(3)$, too]. To get the right quark quantum numbers, we then choose $I_{3,3}=0$, which implies (after fixing the scale) that $I_{3,1}=-I_{3,2}=-1/2$; also we set $Y_1=Y_2=1/3$ and $Y_3=-2/3$. Then we find that, e.g., the name "u_h" is indeed motivated since it now has the right quantum numbers of the physical u quark. The role played by the property "C" is clarified in the next section.

3. MASS GENERATION FROM THE HIERARCHY STATES

We have now found that it is at least consistent to interpret the second level of the hierarchy as corresponding to three quarks, three antiquarks, and the vacuum. Moreover, by demanding that the information content in the hierarchy is maximal, we have achieved an extra dimension compared with $SU(3)$. This "C"-ness is a genuine product of the hierarchy, but we should somehow be able to check whether it is consistent to add to quark quantum numbers a new one.

Let M_0 be a real diagonal 3×3 matrix. We merely note that

$$(\bar{q}_h)^T M_0 q_h=0 \tag{3.1}$$

for every q (T is a transpose). If we now transform the states q_h to physical states q_{phys} by transformation R, with R as before,

$$R=\begin{bmatrix} 1/2 & -1/2 & 0 \\ 1/3 & 1/3 & -2/3 \\ C_1 & C_2 & -(C_1+C_2) \end{bmatrix} \tag{3.2}$$

we get, instead of equation (3.1),

$$\bar{q}^T_{\text{phys}} M_0 q_{\text{phys}} = \bar{q}^T_h R^T M_0 R q_h \equiv m_q \neq 0 \tag{3.3}$$

Therefore we are led to postulate that transformation R, corresponding to renaming of the hierarchy labels, generates masses m_q for the quarks. Let $M_0=(-M_1, -M_2, -M_3)$, where M_k's are real parameters (and the minus signs only a convention). Then from equation (3.3) we get explicitly

$$m_q = I^2_{3,q} M_1 + Y^2_q M_2 + C^2_q M_3 \tag{3.4}$$

where $I_{3,q}$, Y_q, and C_q refer to the quantum numbers of the quark q. From equation (3.4) we get immediately

$$m_d - m_u = \left(C_2^2 - C_1^2\right) M_3 \tag{3.5}$$

which illustrates the role played by the property C. However, these conjectures are not yet testable. But since the mass formula, equation (3.4), depends on at least two *additive* quantum numbers we can, supposing that also C is an additive quantum number, extend this procedure to hadrons.

From $SU(3)$ we know how the hadrons are built up. It is then straightforward to extend equation (3.4) first to baryons. Let a baryon state to be described by $B=q_{\text{phys},1}+q_{\text{phys},2}+q_{\text{phys},3}$. The mass formula for baryons is then by analog

$$m_B^{(k)} = \sum_{q\in B} E_q^{(k)} + \bar{B}^T M_0^{(k)} B \tag{3.6}$$

where $\bar{B}$ is the antibaryon obtained by adding up the respective antiquarks. In the first term on the right-hand side of equation (3.6) we have taken into account the fact that quarks move inside hadrons. $E_q^{(k)}$ can be understood as the kinetic energy contribution of the quark q to the baryon mass. By index k we denote different $SU(3)$ representations; since we have obtained equation (3.6) only by analog, we cannot be sure that the parameters of the matrix M_0 are the same within different representations. We can only demand that $M_0^{(k)}$ is real and diagonal.

From equation (3.6) we get several mass formulas for baryons; for example, for the state $I_3=1/2$, $Y=1$ we get

$$m^{(k)}(1/2,1)=2E_u^{(k)}+E_d^{(k)}+\tfrac{1}{4}M_1^{(k)}+M_2^{(k)}-(2C_1+C_2)^2M_3^{(k)} \quad (3.7)$$

The sign convention for the parameters of $M_0^{(k)}$ is the same as it was with the quarks. Similar formulas as equation (3.7) can of course be written for other states. It is then straightforward to arrive at electromagnetic mass differences like, for example,

$$m(-1/2,1)-m(1/2,1)=E_d-E_u+3(C_2^2-C_1^2)M_3 \quad (3.8)$$

where we have dropped the index k for convenience. Together these differences give the well-known $SU(3)$ result

$$m(\Xi^0)-m(\Xi^-)+m(\Sigma^-)-m(\Sigma^+)=m(n)-m(p) \quad (3.9)$$

Note that equation (3.9) should be satisfied not only by octet masses but also by decuplet masses. Experimentally[1] for decuplet left-hand side (LHS) is ~ -2 MeV and right-hand side (RHS) (where, naturally, n and p are replaced, respectively, by Δ^0 and Δ^+)$\sim -4/3$ MeV, supposing—reasonably we believe—that $m(\Delta^-)-m(\Delta^{++})\sim -4$ MeV.

For decuplet we get also the $SU(3)$ result,

$$3\left[m(\Delta^+)-m(\Delta^0)\right]=m(\Delta^-)-m(\Delta^{++}) \quad (3.10)$$

but *not* the $SU(3)$ result $m(\Sigma^0)-m(\Sigma^-)=m(\Xi^0)-m(\Xi^-)$, which is not well satisfied (experimentally LHS is ~ -0.2 MeV and RHS $= -3.2$ MeV).

As we have seen in equation (3.5), the difference C_1-C_2 measures electromagnetic mass splitting. If we set $C_1 \sim C_2$ (which also implies $E_u \sim E_d$), we can expect to get mean masses of different charge states. Then for decuplet, putting $I_3(\Delta)=3/2$, we obtain

$$3m(\Sigma)+m(\Omega)=m(\Delta)+3m(\Xi) \quad (3.11)$$

which is somewhat better than the usual $SU(3)$ equal mass spacing rule. Experimentally the difference LHS $-$ RHS $=5.8$ MeV. (As one would expect, this formula has been presented earlier in the literature, though derived from totally different premises; see Bisiacchi and Fronsdal, 1966).

However, for octet we do not get any mass formula for mean masses of charge states. Besides, there is no mass difference between Λ and Σ^0.

[1]All experimental values quoted in this paper are from Particle Date Group, 1978.

For mesons there are too many free parameters, and no mass formulas arise. In spite of these shortcomings, we believe that mass relations (3.9)–(3.11) strengthen our interpretation as outlined in Section 2, and that our approach to the interpretation problem of the combinatorial hierarchy is basically sound. Therefore we are left with generalizing the procedure we used in Section 2 to other hierarchy levels. This will be discussed in the next section

4. INTERPRETING OTHER LEVELS

Ideas presented in Section 2 can be extended to other levels. Consider the first level. Bastin and Noyes interpret the three columns $e_1 = \binom{1}{0}$, $e_2 = \binom{0}{1}$, and $e_3 = \binom{1}{1}$ to represent, respectively, charge states $Q = +$, $Q = -$, and $Q = \pm$; they take the first element of the column to correspond to presence or absence of charge $Q = +$ and the second to $Q = -$. However, since one of the main attractions of the combinatorial hierarchy is the appearance of suggestive cumulative sums of DCsS (see Table I), one would expect the appearance of charge to be connected with the third level rather than the first. Moreover, Bastin and Noyes are guided in their interpretation by the belief that charge, baryon number, and lepton number are all absolutely conserved, and of course, experimentally this is well known to be true. However, within grand unified theories there are serious theoretical expectations concerning nonconservation of baryon and lepton numbers (for some reviews of grand unified theories, see Pati, 1978; Fritz and Minkowski, 1975; Georgi and Glashow, 1974.)

To conform with these expectations, we believe *fermion number* to be a more significant quantity than baryon or lepton numbers. Then the first level can easily be interpreted as a fermion and an antifermion, and the column $\binom{1}{1}$ is again representing the vacuum.

Since the weak coupling constant does not arise in the combinatorial hierarchy in the same way as other coupling constants (i.e., as cumulative sums of DCsS), it would be tempting to exclude leptons from the hierarchy classification and consider it as a classification scheme for hadrons only. However, it is fair to remind the reader that these conjectures are very speculative.

Because of the purely fermionic character of the first level, we could say that it is associated with spin. Similarly, the second level, because of its strong interaction character, can be associated with color. The third level has, after the subtraction of the vacuum, 126 particle states. One could try to relate these states either to new particles or to conventional hadrons. (Out of three quarks and antiquarks we can build $27+27+9=63$ baryon and meson states, and with an additional two-valued quantum number we get altogether 126 states). This level should be connected with charge. The fourth level we associate, following Bastin and Noyes, with gravity. Thus every level is, in the absence of dynamics, associated with an unbroken symmetry while the *contents* of levels perhaps describe symmetries that are broken (for fermion number violation in grand unified theories, see Pati et al., 1975; Fritz and Minkowski, 1975).

To justify the connection of the *level* with (unbroken) symmetries and at the same time to illustrate the name aspect of the hierarchy elements, consider, to take an example, a Hilbert space with eight orthogonal state vectors, labeled by $|e_0\rangle, |e_1\rangle, \ldots, |e_7\rangle$, where e_0 is the neutral element and e_k's ($k=1,\ldots,7$) are the columns of Section 2 obeying algebra modulo 2 and forming the second level of the hierarchy. Quantum number conjugation C can then be defined by

$$C|e_k\rangle = |e_k + e_7\rangle = |\bar{e}_k\rangle \tag{4.1a}$$

$$C|\bar{e}_k\rangle = |\bar{e}_k + e_7\rangle = |e_k\rangle \tag{4.1b}$$

But vacuum is not self-conjugate:

$$C|e_7\rangle = |e_0\rangle, \qquad C|e_0\rangle = |e_7\rangle \tag{4.2}$$

Then, in a Hilbert space we must define the physical vacuum as

$$|0\rangle = 1/(2)^{1/2}(|e_0\rangle + |e_7\rangle) \tag{4.3}$$

while the state orthogonal to it is

$$|x\rangle = 1/(2)^{1/2}(|e_0\rangle - |e_7\rangle) \tag{4.4}$$

Here $|x\rangle$ is a state not appearing in the hierarchy, having the conjugation property $C|x\rangle = -|x\rangle$. It would be tempting to interpret this as a quantum associated with, but not belonging to, the second level. The appearance of a similar "quantum" in Hilbert space holds true for every level, but of course this is pure guesswork and the details must be worked out later.

5. DISCUSSION

In conclusion, we have shown that the second level of the combinatorial hierarchy can consistently be interpreted in terms of three quarks, three antiquarks, and the vacuum. Our technique was outlined in Section 2 and further tested for consistency in Section 3. In Section 4 we sketched a possible generalization to other hierarchy levels. For more quarks, one can postulate a second combinatorial hierarchy where there is a new triplet of quarks at the second level, say, c, b, and t, etc.

It is interesting to note that the combinatorial hierarchy may reflect the structure of gauge hierarchies. For example, the breaking of the second level clearly corresponds to breaking of $SO(7)$ or $SO(6)$ to $SU(3)\times U(1)$, as transformation (3.2) maps the representation $\underline{7}$ of $SO(7)$ to the same elements as it does with the second level states. Furthermore, breaking of the third level can then be understood (being probably effectively seven dimensional only; see Bastin et al., 1979) as breaking of 126-dimensional representation of a group of rank 7. This drastically limits possible interpretations. The only anomaly-free choice seems to be representations $\underline{28}+\underline{28}^*+\underline{70}$ of $SU(8)$, which could then be broken into $SU(2)\times U(1)\times SU(5)$, for example.

The fact that the hierarchy at the fourth level explodes into a terrifying number of states (probably at least several, even tens of orders of magnitude) is somewhat disturbing but of course not experimentally excluded.

We did not utilize the knowledge of hierarchy level "coupling constants"; however, *if* they really can be interpreted as physical coupling constants, it is obvious that they should play a central role in constructing hierarchy dynamics. In the Appendix we present a mass formula loosely based on this knowledge, which produces extremely accurately the whole P-wave baryon spectrum.

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APPENDIX: MASS FORMULA FOR BARYONS

Baryon masses are believed to arise from the masses and binding energies of their constituent quarks. However, we do not know the quark masses nor their energies inside hadrons in an exactly calculable form. On the other hand, in meson masses we have some information about interactions between quarks. Thus, if we knew some baryon mass M_0, we could at least try to write another baryon mass M as $M=M_0+f(M_0,\dots)$ where f is some function depending on mass M_0 and on some additional input, say, some meson masses. Moreover, the "coupling constant" of the second level should play some role in a baryon mass formula. However, there is an ambiguity present: should we use the "coupling constant" of the second level (1/10) or the "coupling constant" of the third-dimensional first level (1/7)? This choice is dictated by observed mass values only. The answer turns out to be that we should use $g_0 = 1/7$, when writing our phenomenological mass formula as

$$M(Y \pm 1) = M_0(Y) + \frac{G[M_0(Y)+m]^2}{M_0(Y)} \tag{A.1}$$

where M and M_0 are some baryon masses, Y is the hypercharge, m is the mass of some meson (K, π, ρ, η, or η'), and G is the relevant coupling constant, given by $G = g_0/2^n$, where $g_0 = 1/7$ and $n = 0, 1, \ldots$ indicates that we are dealing with nth radial excitation of the nucleon.

TABLE II. Theoretical P-wave baryon spectrum resulting from equation (A.1) Errors are based on experimental errors of M_0 and m.

Particle	M_{exp} (MeV)	M_{theor} (MeV)	M_0^b	m^b
Λ	1115.60 ± 0.05	1115.14 ± 0.06	N	π
Σ	1193.06 ± 0.12^a	1195.0 ± 0.2	N	$K-\pi$
Ξ	1318.1 ± 0.6^a	1316.8 ± 0.2	Λ	π
$\Sigma(1385)$	1383.9 ± 2.6^a	1386.4 ± 1.3	N	ρ
$\Xi(1533)$	1533.4 ± 0.7^a	1534.5 ± 0.3	Σ	K
$\Omega(1672)$	1672.2 ± 0.4	1674.7 ± 0.7	Ξ	K
$N(1470)$	1390–1470	1470.3 ± 0.6	Λ	η
$n=1$				
$N(1470)$	1390–1470	1470.0 ± 0.4	Σ	η'
$\Lambda(1600)$		1596	$N(1470)$	π
$\Sigma(1660)$	1580–1690	1632	$N(1470)$	$K-\pi$
Σ??	—	1715	$N(1470)$	ρ
Ξ??	—	1731	$\Lambda(1596)$	π
$\Xi(1820)$	1823 ± 6	1830	$\Sigma(1632)$	K
Ω??	—	1935	$\Xi(1730)$??	K
$N(1780)$	1650–1750	1802	$\Lambda(1596)$	η
$n=2$				
$N(1780)$	1650–1750	1779	$\Sigma(1632)$	η'
$\Lambda(1860)$	1850–1920	1854	$N(1780)$	π
$\Sigma(1880)$		1872	$N(1780)$	$K-\pi$
Σ??	—	1911	$N(1780)$	ρ
Ξ??	—	1951	$\Lambda(1860)$	π
Ξ??	—	1979	$\Sigma(1872)$	K
Ω??	—	2060	$\Xi(1951)$?	K
$n=3$				
N??	—	1970	$\Lambda(1860)$	η

[a] Experimental value is taken to be simple mean of different charge states.
[b] As input values of M_0 and m we have used simple means of different charge states.
[c] Not well established.

Starting from the nucleon, we can then generate the whole P-wave baryon spectrum very accurately. The resulting spectrum is presented in Table II. We will mention only some main points concerning the mass formula A.1.

(i) K, π, and ρ generate states M which differ from M_0 by -1 units of hypercharge Y; η and η' generate excited nucleon states which differ from M_0 by $+1$ units of hypercharge.

(ii) Octet Σ/Λ states are somewhat problematic; Σ states are reached from the nucleon with $m = M_K - M_\pi$, and the decuplet states with $Y = 1$ (i.e., Δ states) do not appear with any reasonable combination of masses in equation (A.1).

The connection of the mass formula (A.1) to the combinatorial hierarchy is admittedly weak and rests only on the equality of the hierarchy "coupling constant" and the quantity g_0 appearing in equation (A.1). However, the form of the mass formula (A.1) is simple and its quality is very good—in general, errors relative to experimental values seem to be of the order $10^{-3}-10^{-4}$, depending on which charge states of mesons and baryons are used in the mass formula—and this makes its connection to the combinatorial hierarchy, though perhaps nonexistent, worth investigating.

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